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ORIGINAL PAPER

A robust fuzzy adaptive law for evolving control systems

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Abstract In this paper an adaptive law with leakage is presented. This law can be used in the consequent part of Takagi–Sugeno-based control. The approach enables easy implementation in the control systems with evolving antecedent part. This combination results in a high-performance and robust control of nonlinear and slowly varying systems. It is shown in the paper that the proposed adaptive law is a natural way to cope with the parasitic dynamics. The boundedness of estimated parameters, the tracking error and all the signals in the system is guaranteed if the leakage parameter σ' is large enough. This means that the proposed adaptive law ensures the global stability of the system. A simulation example is given that illustrates the proposed approach.

Keywords Adaptive law · Takagi–Sugeno model · Model-reference control · Evolving systems

1 Introduction

The problem of control of nonlinear plants has received a great deal of attention in the past. The problem itself is fairly demanding, but when the model of the plant is unknown or poorly known, the solution becomes considerably more difficult. Nevertheless, several approaches exist to solve the problem. One possibility is to apply adaptive control. Adaptive control schemes for linear systems do not produce good results, although adaptive parameters try to track the "true" local linear parameters of the current operating point which is done with some lag after each operating-point change. To overcome this problem, adaptive control was extended in the 1980s and 1990s to time-varying and nonlinear plants (Krstić et al. 1995). Since we restricted our attention mainly to nonlinear plants that are very slowly varying, the former approaches were not as relevant even though they produce better results than classical adaptive control. The main drawback of adaptive control algorithms for nonlinear plants is that they demand fairly good knowledge of mathematics and are thus avoided by practising engineers.

Many successful applications of fuzzy controllers (Precup and Hellendoorn 2011; Galichet and Foulloy 2003; Precup et al. 2003) have shown their ability to control nonlinear plants. A possible extension is to introduce some sort of adaptation into the fuzzy controller. The first attempts at constructing a fuzzy adaptive controller can be traced back to Procyk and Mamdani (1979), where so-called linguistic self-organising controllers were introduced. Many approaches were later presented where a fuzzy model of the plant was constructed on-line, followed by control parameters adjustment (Layne and Passino 1993). The main drawback of these schemes was that their stability was not treated rigorously. The universal approximation theorem (Wang and Mendel 1992) provided a theoretical background for new fuzzy direct and indirect adaptive controllers (Wang and Mendel 1992; Tang et al. 1999; Pomares et al. 2002; Rojas et al. 2006; Vaščák 2012; Johanyák and Papp 2012; Precup et al. 2012) whose stability was proven using the Lyapunov theory.

Robust adaptive control was proposed to overcome the problem of disturbances and unmodeled dynamics (Ioannou and Sun 1996). Similar solutions have also been used in adaptive fuzzy and neural controllers, i.e. projection (Tong et al. 2000), dead zone (Koo 2001), leakage (Ge and

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Wang 2002), adaptive fuzzy backstepping control (Tong and Li 2012) etc. have been included in the adaptive law to prevent instability due to reconstruction error.

The control of a practically very important class of plants is treated in the paper that, in our opinion, occur quite often in process industries. The class of plants consists of nonlinear systems of arbitrary order but where the control law is based on the first-order nonlinear approximation. The dynamics not included in the first-order approximation are referred to as parasitic dynamics. The parasitic dynamics are treated explicitly in the development of the adaptive law to prevent the modelling error to grow unbounded. The class of plant also includes bounded disturbances.

The choice of simple nominal model results in very simple control and adaptive laws. The control law is similar to the one proposed by Blažič et al. (2003, 2012) but an extra term is added in the current paper. In this paper a novel adaptive law with leakage will be presented. It will be shown in the paper that the proposed adaptive law is a natural way to cope with parasitic dynamics. The boundedness of estimated parameters, the tracking error and all the signals in the system will be proven if the leakage parameter σ' satisfies certain condition. This means that the proposed adaptive law ensures the global stability of the system. A very important property of the proposed approach is that it can be used in the consequent part of Takagi–Sugeno-based control. The approach enables easy implementation in the control systems with evolving antecedent part (Angelov et al. 2001, 2011; Angelov and Filev 2004; Cara et al. 2010; Sadeghi-Tehran et al. 2012). This combination results in a high-performance and robust control of nonlinear and slowly varying systems.

2 The class of nonlinear plants

Our goal is to design control for a class of plants that include nonlinear time-invariant systems where the model behaves similarly to a first-order system at low frequencies (the frequency response is not defined for nonlinear systems so frequencies are meant here in a broader sense). If the plant were the first-order system (without parasitic dynamics), it could be described by a fuzzy model in the form of if-then rules:

$$\begin{aligned} &\textbf{if } z_1 \textbf{ is } A_{i_a} \textbf{ and } z_2 \textbf{ is } B_{i_b} \textbf{ then } \dot{y}_p = -a_i y_p + b_i u + c_i \\ &i_a = 1, \ldots, n_a \quad i_b = 1, \ldots, n_b \quad i = 1, \ldots, k \end{aligned} \tag{1}$$

where u and y_p are the input and the output of the plant respectively, A_{i_a} and B_{i_b} are fuzzy membership functions, and a_i, b_i, and c_i are the plant parameters in the i-th domain. Note the c_i term in the consequent. Such an additive term is obtained if a nonlinear system is linearised in an operating point. This additive term changes by changing the operating point. The term c_i is new comparing to the model used in (Blažič et al. 2003, 2012). The antecedent variables that define the domain in which the system is currently situated are denoted by z_1 and z_2 (actually there can be only one such variable or there can also be more of them, but this does not affect the approach described in this paper). There are n_a and n_b membership functions for the first and the second antecedent variables, respectively. The product $k = n_a \times n_b$ defines the number of fuzzy rules. The membership functions have to cover the whole operating area of the system. The output of the Takagi–Sugeno model is then given by the following equation

$$\dot{y}_p = \frac{\sum_{i=1}^{k} \left[\beta_i^0(\varphi)(-a_i y_p + b_i u + c_i)\right]}{\sum_{i=1}^{k} \beta_i^0(\varphi)} \tag{2}$$

where φ represents the vector of antecedent variables z_i (in the case of fuzzy model given by Eq. (1), $\varphi = [z_1 \;\; z_2]^T$). The degree of fulfilment $\beta_i^0(\varphi)$ is obtained using the T-norm, which in this case is a simple algebraic product of membership functions

$$\beta_i^0(\varphi) = T(\mu_{A_{i_a}}(z_1), \mu_{B_{i_b}}(z_2)) = \mu_{A_{i_a}}(z_1) \cdot \mu_{B_{i_b}}(z_2) \tag{3}$$

where $\mu_{A_{i_a}}(z_1)$ and $\mu_{B_{i_b}}(z_2)$ stand for degrees of fulfilment of the corresponding fuzzy rule. The degrees of fulfilment for the whole set of fuzzy rules can be written in a compact form as

$$\beta^0 = \left[\, \beta_1^0 \beta_2^0 \ldots \beta_k^0 \right]^T \in \mathbb{R}^k \tag{4}$$

or in a more convenient normalised form

$$\beta = \frac{\beta^0}{\sum_{i=1}^{k} \beta_i^0} \in \mathbb{R}^k \tag{5}$$

Due to (2) and (5), the first-order plant can be modelled in fuzzy form as

$$\dot{y}_p = -(\beta^T \mathbf{a}) y_p + (\beta^T \mathbf{b}) u + (\beta^T \mathbf{c}) \tag{6}$$

where $\mathbf{a} = [a_1 a_2 \ldots a_k]^T, \mathbf{b} = [b_1 b_2 \cdots b_k]^T$, and $\mathbf{c} = [c_1 c_2 \ldots c_k]^T$ are vectors of unknown plant parameters in respective domains ($\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^k$).

To assume that the controlled system is of the first order is a quite huge idealisation. Parasitic dynamics and disturbances are therefore included in the model of the plant. The fuzzy model of the first order is generalised by adding stable factor plant perturbations and disturbances, which results in the following model (Blažič et al. 2003):

$$\begin{aligned} \dot{y}_p(t) = &-(\beta^T(t)\mathbf{a}) y_p(t) + (\beta^T(t)\mathbf{b}) u(t) + (\beta^T \mathbf{c}) \\ &- \Delta_y(p) y_p(t) + \Delta_u(p) u(t) + d(t) \end{aligned} \tag{7}$$

where p is a differential operator d/dt, $\Delta_y(p)$ and $\Delta_u(p)$ are stable strictly proper linear operators, while d is bounded signal due to disturbances (Blažič et al. 2003).

Equation (7) represents the class of plants to be controlled by the approach proposed in the following sections. The control is designed based on the model given by Eq. (6) while the robustness properties of the algorithm prevent the instability due to parasitic dynamics and disturbances.

3 The proposed fuzzy adaptive control algorithm

A fuzzy model reference adaptive control is proposed in the paper to achieve tracking control for the class of plants described in the previous section. The control goal is that the plant output follows the output y_m of the reference model. The latter is defined by a first order linear system $G_m(p)$:

$$y_m(t) = G_m(p)w(t) = \frac{b_m}{p + a_m} w(t) \tag{8}$$

where $w(t)$ is the reference signal while b_m and a_m are the constants that define desired behaviour of the closed system. The tracking error

$$\varepsilon(t) = y_p(t) - y_m(t) \tag{9}$$

therefore represents some measure of the control quality. To solve the control problem simple control and adaptive laws are proposed in the following subsections.

3.1 Control law

The control law is very similar to the one proposed by Blažič et al. (2003, 2012):

$$u(t) = \left(\beta^T(t)\hat{\mathbf{f}}(t)\right)w(t) - \left(\beta^T(t)\hat{\mathbf{q}}(t)\right)y_p(t) + \left(\beta^T(t)\hat{\mathbf{r}}(t)\right) \tag{10}$$

where $\hat{\mathbf{f}}(t) \in \mathbb{R}^k, \hat{\mathbf{q}}(t) \in \mathbb{R}^k$, and $\hat{\mathbf{r}}(t) \in \mathbb{R}^k$ are the control gain vectors to be determined by the adaptive law. This control law is obtained by generalising the model reference adaptive control algorithm for the first order linear plant to the fuzzy case. The control law also includes the third term that is new with respect to the one in Blažič et al. (2012). It is used to compensate the $(\beta^T\mathbf{c})$ term in Eq. (7).

3.2 Adaptive law

The adaptive law proposed in this paper is based on the adaptive law from Blažič et al. (2003). The e_1-modification was used in the leakage term in Blažič et al. (2003). An alternative approach was proposed in Blažič et al. (2012) where quadratic term is used the leakage. But a new adaptive law for $\hat{r}_i$ is also proposed here:

$$\begin{aligned}
\dot{\hat{f}}_i &= -\gamma_{fi} b_{sign} \varepsilon w \beta_i - \gamma_{fi}\sigma' w^2 \beta_i^2 (\hat{f}_i - \hat{f}_i^*) \quad i = 1, 2, \ldots k \\
\dot{\hat{q}}_i &= \gamma_{qi} b_{sign} \varepsilon y_p \beta_i - \gamma_{qi}\sigma' y_p^2 \beta_i^2 (\hat{q}_i - \hat{q}_i^*) \quad i = 1, 2, \ldots k \\
\dot{\hat{r}}_i &= -\gamma_{ri} b_{sign} \varepsilon \beta_i - \gamma_{ri}\sigma' \beta_i^2 (\hat{r}_i - \hat{r}_i^*) \quad i = 1, 2, \ldots k
\end{aligned} \tag{11}$$

where γ_{fi}, γ_{qi}, and γ_{ri} are positive scalars referred to as adaptive gains, $\sigma' > 0$ is the parameter of the leakage term, $\hat{f}_i^*, \hat{q}_i^*$, and $\hat{r}_i^*$ are the a priori estimates of the control gains $\hat{f}_i, \hat{q}_i$, and $\hat{r}_i$ respectively, and b_{sign} is defined as follows:

$$b_{sign} = \begin{cases} 1 & b_1 > 0, \quad b_2 > 0, \ldots b_k > 0 \\ -1 & b_1 < 0, \quad b_2 < 0, \ldots b_k < 0 \end{cases} \tag{12}$$

If the signs of all elements in vector $\mathbf{b}$ are not the same, the plant is not controllable for some β ($\beta^T\mathbf{b}$ is equal to 0 for this β) and any control signal does not have an effect.

It is possible to rewrite the adaptive law (11) in the compact form if the control gain vectors $\hat{\mathbf{f}}, \hat{\mathbf{q}}$, and $\hat{\mathbf{r}}$ are defined as

$$\begin{aligned}
\hat{\mathbf{f}}^T &= [\hat{f}_1\ \hat{f}_2 \ldots \hat{f}_k] \\
\hat{\mathbf{q}}^T &= [\hat{q}_1\ \hat{q}_2 \ldots \hat{q}_k] \\
\hat{\mathbf{r}}^T &= [\hat{r}_1\ \hat{r}_2 \ldots \hat{r}_k]
\end{aligned} \tag{13}$$

Then the adaptive law (11) takes the following form:

$$\begin{aligned}
\dot{\hat{\mathbf{f}}} &= -\Gamma_f b_{sign}\, \varepsilon w \beta - \Gamma_f \sigma' w^2 \operatorname{diag}(\beta)\operatorname{diag}(\beta)(\hat{\mathbf{f}} - \hat{\mathbf{f}}^*) \\
\dot{\hat{\mathbf{q}}} &= \Gamma_q b_{sign}\, \varepsilon y_p \beta - \Gamma_q \sigma' y_p^2 \operatorname{diag}(\beta)\operatorname{diag}(\beta)(\hat{\mathbf{q}} - \hat{\mathbf{q}}^*) \\
\dot{\hat{\mathbf{r}}} &= -\Gamma_r b_{sign}\, \varepsilon \beta - \Gamma_r \sigma' \operatorname{diag}(\beta)\operatorname{diag}(\beta)(\hat{\mathbf{r}} - \hat{\mathbf{r}}^*)
\end{aligned} \tag{14}$$

where $\Gamma_f \in \mathbb{R}^{k\times k}, \Gamma_q \in \mathbb{R}^{k\times k}$, and $\Gamma_r \in \mathbb{R}^{k\times k}$ are positive definite matrices, $\operatorname{diag}(\mathbf{x}) \in \mathbb{R}^{k\times k}$ is a diagonal matrix with the elements of vector $\mathbf{x}$ on the main diagonal, while $\hat{\mathbf{f}}^* \in \mathbb{R}^k, \hat{\mathbf{q}}^* \in \mathbb{R}^k$, and $\hat{\mathbf{r}}^* \in \mathbb{R}^k$ are the a priori estimates of the control gain vectors.

3.3 The sketch of the stability proof

The reference model (8) can be rewritten in the following form:

$$\dot{y}_m = -a_m y_m + b_m w \tag{15}$$

By subtracting (15) from (7), the following tracking-error model is obtained

$$\begin{aligned}
\dot{\varepsilon} = -a_m\varepsilon &+ \left[(\beta^T\mathbf{b})(\beta^T\hat{\mathbf{f}}) - b_m\right]w \\
&- \left[(\beta^T\mathbf{b})(\beta^T\hat{\mathbf{q}}) + (\beta^T\mathbf{a}) - a_m\right]y_p \\
&+ \left[(\beta^T\mathbf{b})(\beta^T\hat{\mathbf{r}}) + (\beta^T\mathbf{c})\right] + \Delta_u(p)u - \Delta_y(p)y_p + d
\end{aligned} \tag{16}$$

Now we assume that there exist constant control parameters $\mathbf{f}^*, \mathbf{q}^*$, and $\mathbf{r}^*$ that stabilise the closed-loop system. This is a mild assumption and it is always fulfilled unless the unmodeled dynamics are unacceptably high. These parameters are only needed in the stability analysis and can be chosen to make the "diference" between the closed-loop system and the reference model small in some sense (the defintion of this "diference" is not important for the analysis). The parameters $\mathbf{f}^*, \mathbf{q}^*$, and $\mathbf{r}^*$ are sometimes called the "true" parameters because they result in the perfect tracking in the absence of unmodeled dynamics and disturbances. The parameter errors are defined as:

$$\begin{aligned}\tilde{\mathbf{f}} &= \hat{\mathbf{f}} - \mathbf{f}^* \\ \tilde{\mathbf{q}} &= \hat{\mathbf{q}} - \mathbf{q}^* \\ \tilde{\mathbf{r}} &= \hat{\mathbf{r}} - \mathbf{r}^*\end{aligned} \tag{17}$$

The expressions in the square brackets in Eq. (16) can be rewritten similarly as in Blažič et al. (2003):

$$\begin{aligned}\left[(\beta^T\mathbf{b})(\beta^T\hat{\mathbf{f}}) - b_m\right] &= b_{sign}\beta^T\tilde{\mathbf{f}} + \eta_f \\ &= b_{sign}\sum_{i=1}^{k}\beta_i\tilde{f}_i + \eta_f \\ \left[(\beta^T\mathbf{b})(\beta^T\hat{\mathbf{q}}) + (\beta^T\mathbf{a}) - a_m\right] &= b_{sign}\beta^T\tilde{\mathbf{q}} + \eta_q \\ &= b_{sign}\sum_{i=1}^{k}\beta_i\tilde{q}_i + \eta_q \\ \left[(\beta^T\mathbf{b})(\beta^T\hat{\mathbf{r}}) + (\beta^T\mathbf{c})\right] &= b_{sign}\beta^T\tilde{\mathbf{r}} + \eta_r \\ &= b_{sign}\sum_{i=1}^{k}\beta_i\tilde{r}_i + \eta_r\end{aligned} \tag{18}$$

where bounded residuals $\eta_f(t)$, $\eta_q(t)$, and $\eta_r(t)$ are introduced [the boundedness can be shown simply; see also Blažič et al. (2003)]. The following Lyapunov function is proposed for the proof of stability:

$$V = \frac{1}{2}\varepsilon^2 + \frac{1}{2}\sum_{i=1}^{k}\gamma_{fi}^{-1}\tilde{f}_i^2 + \frac{1}{2}\sum_{i=1}^{k}\gamma_{qi}^{-1}\tilde{q}_i^2 + \frac{1}{2}\sum_{i=1}^{k}\gamma_{ri}^{-1}\tilde{r}_i^2 \tag{19}$$

Calculating the derivative of the Lyapunov function along the solution of the system (16) and taking into account (18) and adaptive laws (11) we obtain:

$$\begin{aligned}\dot{V} &= \varepsilon\dot{\varepsilon} + \sum_{i=1}^{k}\gamma_{fi}^{-1}\tilde{f}_i\dot{\tilde{f}}_i + \sum_{i=1}^{k}\gamma_{qi}^{-1}\tilde{q}_i\dot{\tilde{q}}_i + \sum_{i=1}^{k}\gamma_{ri}^{-1}\tilde{r}_i\dot{\tilde{r}}_i \\ &= -a_m\varepsilon^2 + \eta_f w\varepsilon - \eta_q y_p\varepsilon + \eta_r\varepsilon + \varepsilon\Delta_u(p)u - \varepsilon\Delta_y(p)y_p + \varepsilon d \\ &\quad - \sum_{i=1}^{k}\sigma' w^2\beta_i^2(\hat{f}_i - \hat{f}_i^*)\tilde{f}_i - \sum_{i=1}^{k}\sigma' y_p^2\beta_i^2(\hat{q}_i - \hat{q}_i^*)\tilde{q}_i \\ &\quad - \sum_{i=1}^{k}\sigma'\beta_i^2(\hat{r}_i - \hat{r}_i^*)\tilde{r}_i\end{aligned} \tag{20}$$

In principle the first term on the right-hand side of Eq. (20) is used to compensate for the next six terms while the last three terms prevent parameter drift. The terms from the second one to the seventh one are formed as a product between the tracking error $\varepsilon(t)$ and a combined error $E(t)$ defined as:

$$\begin{aligned}E(t) &= \eta_f(t)w(t) - \eta_q(t)y_p(t) + \eta_r(t) + \Delta_u(p)u(t) \\ &\quad - \Delta_y(p)y_p(t) + d(t)\end{aligned} \tag{21}$$

Eq. (20) can be rewritten as:

$$\begin{aligned}\dot{V} &= -a_m\left(\varepsilon^2 - \frac{E\varepsilon}{a_m}\right) - \sum_{i=1}^{k}\sigma' w^2\beta_i^2(\hat{f}_i - \hat{f}_i^*)\tilde{f}_i \\ &\quad - \sum_{i=1}^{k}\sigma' y_p^2\beta_i^2(\hat{q}_i - \hat{q}_i^*)\tilde{q}_i - \sum_{i=1}^{k}\sigma'\beta_i^2(\hat{r}_i - \hat{r}_i^*)\tilde{r}_i\end{aligned} \tag{22}$$

The first term on the right-hand side of Eq. (22) becomes negative if $|\varepsilon| > \frac{|E|}{a_m}$. If the combined error were a priori bounded, the boundedness of the tracking error ε would be more or less proven. The problem lies in the fact that not only bounded signals ($w(t)$, $\eta_f(t)$, $\eta_q(t)$, $\eta_r(t)$, $d(t)$) are included in $E(t)$, but also the ones whose boundedness is yet to be proven ($u(t)$, $y_p(t)$). If the system becomes unstable, the plant output $y_p(t)$ becomes unbounded and, consequently, the same applies to the control input $u(t)$. If $y_p(t)$ is bounded, it is easy to see from the control law that $u(t)$ is also bounded. Unboundedness of $y_p(t)$ is prevented by leakage terms in the adaptive law. In the last three terms in Eq. (22) that are due to the leakage there are three similar expressions. They have the following form:

$$(\hat{f}_i(t) - \hat{f}_i^*)\tilde{f}_i(t) = (\hat{f}_i(t) - \hat{f}_i^*)(\hat{f}_i(t) - f_i^*) \tag{23}$$

It is simple to see that this expression is positive if either $\hat{f}_i > \max\{\hat{f}_i^*, f_i^*\}$ or $\hat{f}_i < \min\{\hat{f}_i^*, f_i^*\}$. The same reasoning applies to $\hat{q}_i$ and $\hat{r}_i$. This means that the last three terms in Eq. (22) become negative if the estimated parameters are large (or small) enough. The novelty of the proposed adaptive law with respect to the one in Blažič et al. (2003) is in the quadratic terms with y_p and w in the leakage. These terms are used to help cancelling the contribution of εE in (22):

$$\varepsilon E = \varepsilon\eta_f w - \varepsilon\eta_q y_p + \varepsilon\eta_r + \varepsilon\Delta_u(p)u - \varepsilon\Delta_y(p)y_p + \varepsilon d \tag{24}$$

Since $\varepsilon(t)$ is the difference between $y_p(t)$ and $y_m(t)$ and the latter is bounded, $\varepsilon = O(y_p)$ when y_p tends to infinity. By analysing the control law and taking into account stability of parasitic dynamics $\Delta_u(s)$ and $\Delta_y(s)$ the following can be concluded:

$$u = O(y_p),\ \Delta_u(p)u = O(y_p) \Rightarrow \varepsilon E = O(y_p^2) \tag{25}$$

The third term on the right-hand side of Eq. (22) is $-(\hat{q}_i - \hat{q}_i^*)\tilde{q}_i O(y_p^2)$ which means that the "gain" $(\hat{q}_i - \hat{q}_i^*)\tilde{q}_i$ with respect to y_p^2 of the negative contributions to $\dot{V}$ can always become greater (as a result of adaptation) than the fixed gain of quadratic terms with y_p in Eq. (24). The growth of the estimated parameters is also problematic because these parameters are control gains and high gains can induce instability in combination with parasitic dynamics. Consequently, σ' has to be chosen large enough to prevent this type of instability. Note that the stabilisation in the presence of parasitic dynamics is achieved without using an explicit dynamic normalisation that was used in Blažič et al. (2003).

The stability analysis of a similar adaptive law for linear systems was treated in Blažič et al. (2010) where it was proven that all the signals in the system are bounded and the tracking error converges to a residual set whose size depends on the modelling error if the leakage parameter σ' is chosen large enough with respect to the norm of parasitic dynamics. In this paper the "modelling error" is $E(t)$ from Eq. (21), and therefore the residual-set size depends on the size of the norm of the transfer functions $||\Delta_u||$ and $||\Delta_y||$, the size of the disturbance d, and the size of the bounded residuals $\eta_f(t)$, $\eta_q(t)$, and $\eta_r(t)$.

Only the adaptation of the consequent part of the fuzzy rules is treated in this paper. The stability of the system is guaranteed for any (fixed) shape of the membership functions in the antecedent part. This means that this approach is very easy to combine with existing evolving approaches for the antecedent part. If the membership functions are slowly evolving, these changes introduce another term to $\dot{V}$ which can be shown not to be larger than $O(y_p^2)$. This means that the system stability is preserved by the robustness properties of the adaptive laws. If, however, fast changes of the membership functions occur, a rigorous stability analysis would have to be performed.

4 Simulation examples

A simulation example will be given that illustrates the proposed approach. A simulated plant was chosen since it is easier to make the same operating conditions than it would be when testing on a real plant. The simulated test plant consisted of three water tanks. The schematic representation of the plant is given in Fig. 1. The control objective was to maintain the water level in the third tank by changing the inflow into the first tank.

When modelling the plant, it was assumed that the flow through the valve was proportional to the square root of the pressure difference on the valve. The mass conservation equations for the three tanks are:

$$\begin{aligned} S_1\dot{h}_1 &= \phi_{in} - k_1 \mathrm{sign}(h_1 - h_2)\sqrt{|h_1 - h_2|} \\ S_2\dot{h}_2 &= k_1 \mathrm{sign}(h_1 - h_2)\sqrt{|h_1 - h_2|} - k_2 \mathrm{sign}(h_2 - h_3)\sqrt{|h_2 - h_3|} \\ S_3\dot{h}_3 &= k_2 \mathrm{sign}(h_2 - h_3)\sqrt{|h_2 - h_3|} - k_3 \mathrm{sign}(h_3)\sqrt{|h_3|} \end{aligned} \tag{26}$$

where ϕ_{in} is the volume inflow into the first tank, h_1, h_2, and h_3 are the water levels in three tanks, S_1, S_2, and S_3 are areas of the tanks cross-sections, and k_1, k_2, and k_3 are coefficients of the valves. The following values were chosen for the parameters of the system:

$$\begin{aligned} S_1 &= S_2 = S_3 = 2 \times 10^{-2}\,\mathrm{m}^2 \\ k_1 &= k_2 = k_3 = 2 \times 10^{-4}\mathrm{m}^{5/2}\mathrm{s}^{-1} \end{aligned} \tag{27}$$

The nominal value of inflow ϕ_{in} was set to $8 \times 10^{-5}\mathrm{m}^3\mathrm{s}^{-1}$, resulting in steady-state values 0.48, 0.32 and 0.16 m for h_1, h_2, and h_3 respectively. In the following, u and y_p denote deviations of ϕ_{in} and h_3 respectively from the operating point.

By analysing the plant it can be seen that the plant is nonlinear. It has to be pointed out that the parasitic dynamics are also nonlinear, not just the dominant part as was assumed in deriving the control algorithm. This means that this example will also test the ability of the proposed control to cope with nonlinear parasitic dynamics. The coefficients of the linearised system in different operating points depend on u, h_1, h_2, and h_3 even though that only y_p will be used as an antecedent variable z_1 which is again violation of the basic assumptions but still produces fairly good results.

The proposed control algorithm was compared to a classical model reference adaptive control (MRAC) with e_1-

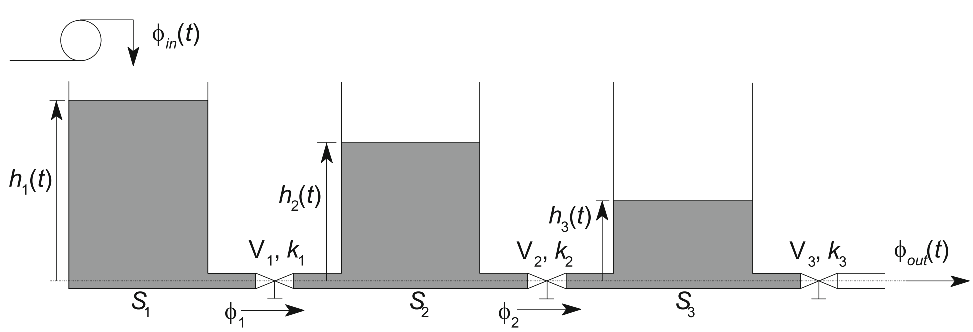

Fig. 1 Schematic representation of the plant

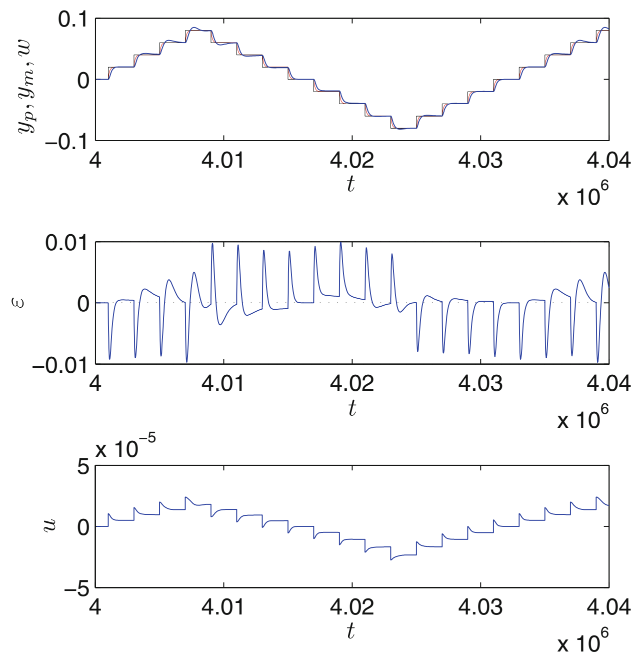

Fig. 2 The MRAC controller—time plots of the reference signal and outputs of the plant and the reference model (*upper figure*), time plot of tracking error (*middle figure*), and time plot of the control signal (*lower figure*)

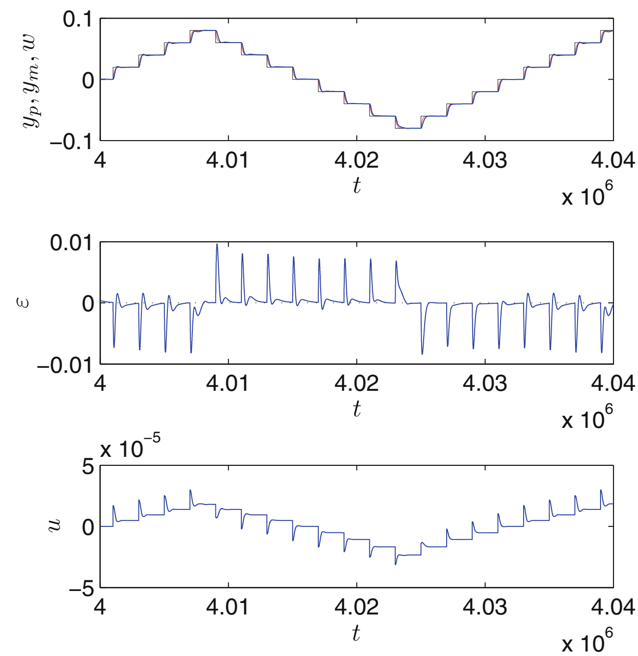

Fig. 3 The proposed approach—time plots of the reference signal and outputs of the plant and the reference model (*upper figure*), time plot of tracking error (*middle figure*), and time plot of the control signal (*lower figure*)

modification. Adaptive gains γ_{fi}, γ_{qi}, and γ_{ri} in the case of the proposed approach were the same as γ_f, γ_q, and γ_r, respectively, in the case of MRAC. A reference signal was chosen as a periodic piece-wise constant function which covered quite a wide area around the operating point (± 50 % of the nominal value). There were 11 triangular fuzzy membership functions (the fuzzification variable was y_p) used; these were distributed evenly across the interval $[-0.1, 0.1]$. As already said, the evolving of the antecedent part was not done in this work. The control input signal u was saturated at the interval $[-8 \times 10^{-5}, 8 \times 10^{-5}]$. No prior knowledge of the estimated parameters was available to us, so the initial parameter estimates were 0 for all examples.

The design objective is that the output of the plant follows the output of the reference model $0.01/(s + 0.01)$. The reference signal was the same in all cases. It consisted of a periodic signal. The results of the experiment with the classical MRAC controller with e_1-modification are shown in Fig. 2.

We used the following design parameters: $\gamma_f = 10^{-4}, \gamma_q = 2 \times 10^{-4}, \gamma_r = 10^{-6}, \sigma' = 0.1$. Figures 3 and 4 show the results of the proposed approach, the former shows a period of system responses after the adaptation has settled, the latter depicts time plots of the estimated parameters. Since $\hat{\mathbf{f}}$, $\hat{\mathbf{q}}$, and $\hat{\mathbf{r}}$ are vectors, all elements of the vectors are depicted. Note that every change in the reference signal results in a sudden increase in tracking error ε (up to 0.01). This is due to the fact that zero tracking of the reference model with relative degree 1 is not possible if the plant has relative degree 3.

The experiments show that the performance of the proposed approach is better than the performance of the MRAC controller for linear plant which is expectable due to nonlinearity of the plant. Very good results are obtained in the case of the proposed approach even though that the parasitic dynamics are nonlinear and linearised parameters depend not only on the antecedent variable y_p but also on others. The spikes on ε in Fig. 3 are consequences of the fact that the plant of 'relative degree' 3 is forced to follow the reference model of relative degree 1. These spikes are inevitable no matter which controller is used.

The drawback of the proposed approach is relatively slow convergence since the parameters are only adapted when the corresponding membership is non-zero. This drawback can be overcome by using classical MRAC in the beginning when there are no parameter estimates or the estimates are bad. When the system approaches desired behaviour the adaptation can switch to the proposed one by initialising all elements of vectors $\hat{\mathbf{f}}$, $\hat{\mathbf{q}}$, and $\hat{\mathbf{r}}$ with estimated scalar parameters from the classical MRAC.

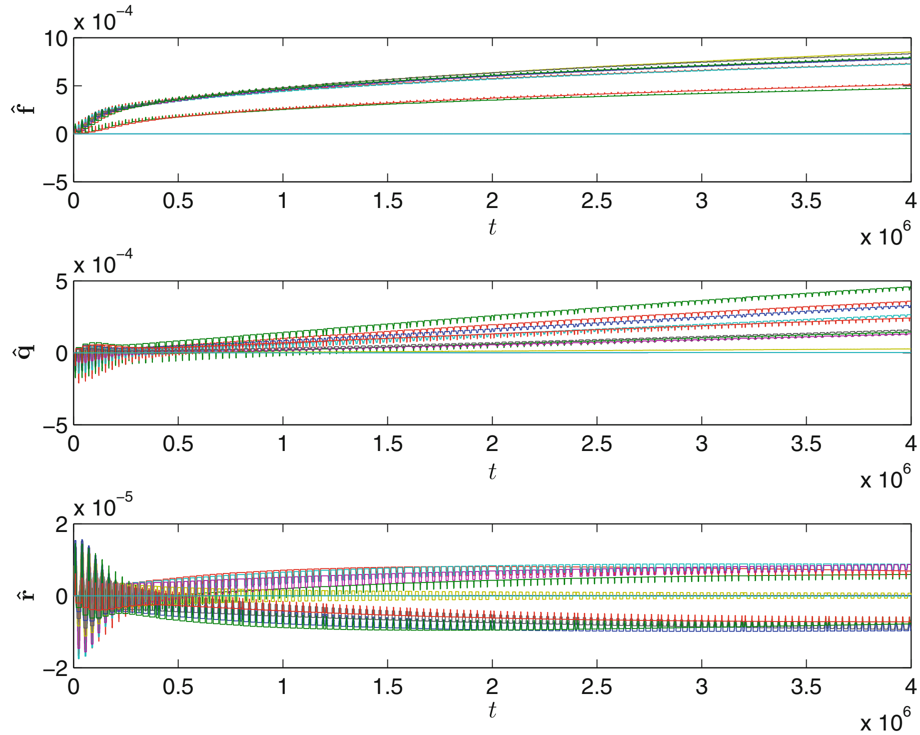

Fig. 4 The proposed approach—time plots of the control gains

5 Conclusion

A new control approach comprising of the known control law and a new adaptive law was presented in the paper. The advantage of the proposed approach is that it is very simple to design but it still offers the advantages of nonlinear and adaptive controllers. The approach enables easy implementation in control systems with evolving antecedent part. This combination results in a high-performance and robust control of nonlinear and slowly varying systems. It was shown on the example that good results can be obtained if a third order plant is treated as a first order plant. The drawback of the approach is long time of adaptation that is the result of the large number of parameters that have to be estimated. In some cases this is also an advantage—if the disturbance is present in some domains, only the corresponding fuzzy control gains will be affected.

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